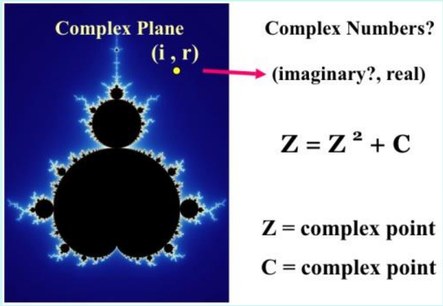
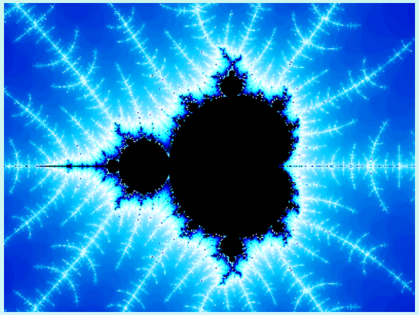
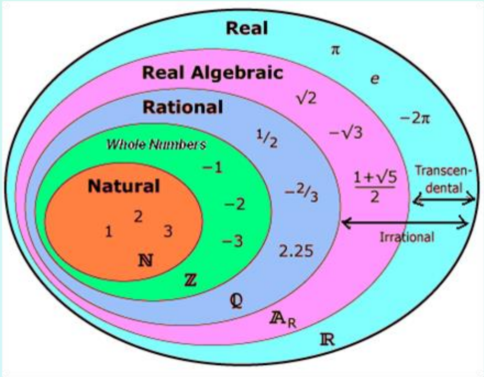
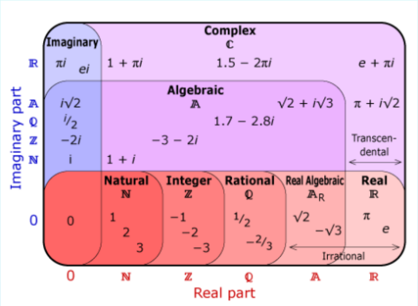
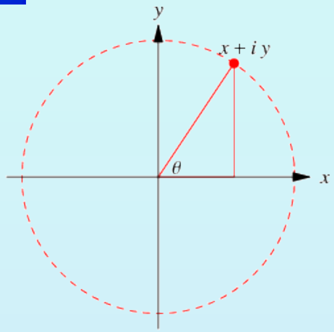


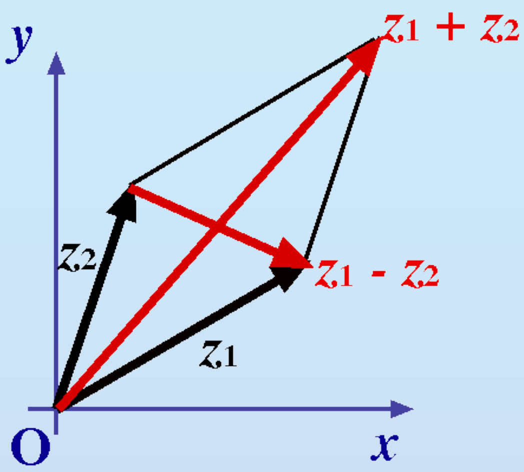


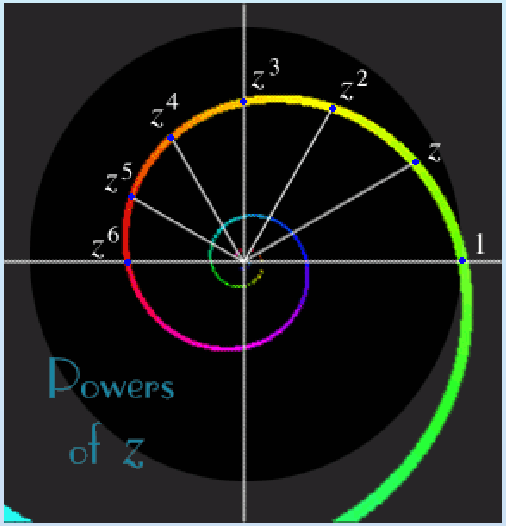


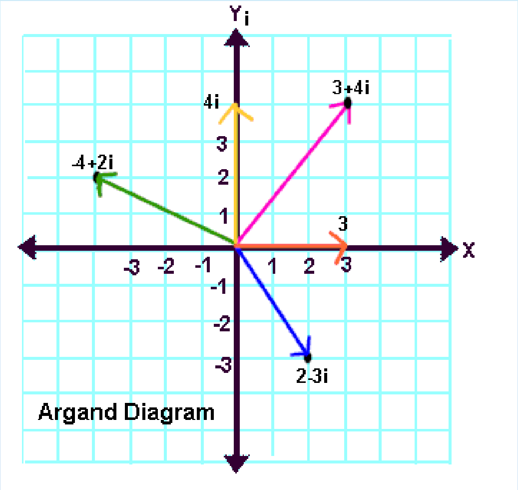




Complex Numbers

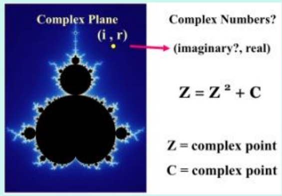






Introduction

- The first chapter of FP1 introduces you to imaginary and complex numbers
- You will have seen at GCSE level that some quadratic equations cannot be solved
- Imaginary and complex numbers will allow us to actually solve these equations!
- We will also see how to represent them on an Argand diagram
- We will also see how to use complex numbers to solve cubic and quartic equations



Complex Numbers

You can use both real and imaginary numbers to solve equations

At GCSE level you met the Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The part under the square root sign is known as the 'discriminant', and can be used to determine how many solutions the equation has:

$$b^2 - 4ac > 0 \quad \longrightarrow \ 2\ real\ roots$$

$$b^2 - 4ac = 0 \quad \longrightarrow 1\ real\ root$$

$$b^2 - 4ac < 0 \quad \longrightarrow 0\ real\ roots$$

The problem is that we cannot square root a negative number, hence the lack of real roots in the 3rd case above

To solve these equations, we can use the imaginary number 'i'

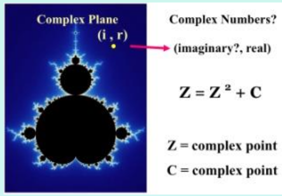
$$i = \sqrt{-1}$$

The imaginary number 'i' can be combined with real numbers to create 'complex numbers'

An example of a complex number would be:

$$5 + 2i$$

Complex numbers can be added, subtracted, multiplied and divided in the same way you would with an algebraic expression



Complex Numbers

You can use both real and imaginary numbers to solve equations

1) Write √-36 in terms of i

This sign means the **positive** square root →

$$\sqrt{-36}$$

Split up using surd manipulation

$$\sqrt{36}\sqrt{-1}$$

Simplify each part
→ √-1 = i

$$= 6i$$

2) Write √-28 in terms of i

$$\sqrt{-28}$$

Split up into a positive and negative part

$$\sqrt{28}\sqrt{-1}$$

Split up the 28 further…

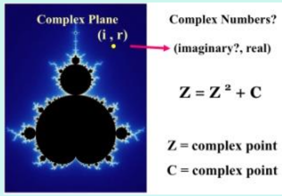
$$\sqrt{4}\sqrt{7}\sqrt{-1}$$

Simplify each part

$$= 2\sqrt{7}i$$

This is usually written in this way

$$= 2i\sqrt{7}$$



Complex Numbers

You can use both real and imaginary numbers to solve equations

Solve the equation:

$$x^2 + 9 = 0$$

$$x^2 + 9 = 0$$

Subtract 9

$$x^2 = -9$$

Square root – we need to consider both positive and negative as we are solving an equation

$$x = \pm\sqrt{-9}$$

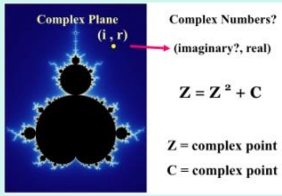
Split up

$$x = \pm\sqrt{9}\sqrt{-1}$$

Write in terms of i

$$x = \pm 3i$$

You should ensure you write full workings – once you have had a lot of practice you can do more in your head!



Complex Numbers

You can use both real and imaginary numbers to solve equations

Solve the equation:

$$x^2 + 6x + 25 = 0$$

→ You can use one of two methods for this

→ Either 'Completing the square' or the Quadratic formula

$$(x+3)^2$$

Imagine squaring the bracket

$$(x+3)(x+3)$$

This is the answer we get

$$x^2 + 6x + 9$$

The squared bracket gives us both the x^2 term and the 6x term
→ It only gives us a number of 9, whereas we need 25 – add 16 on!

<u>Completing the square</u>

$$x^2 + 6x + 25 = 0$$

Write a squared bracket, with the number inside being half the x-coefficient

$$(x+3)^2 + 16 = 0$$

Subtract 16

$$(x+3)^2 = -16$$

Square root

$$x + 3 = \pm\sqrt{-16}$$

Subtract 3

$$x = -3 \pm \sqrt{-16}$$

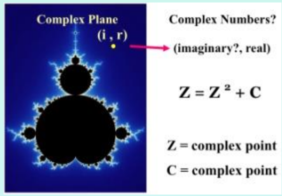
Split the root up

$$x = -3 \pm \sqrt{16}\sqrt{-1}$$

Simplify

$$x = -3 \pm 4i$$

If the x term is even, and there is only a single x^2, then completing the square will probably be the quickest method!



Complex Numbers

You can use both real and imaginary numbers to solve equations

Solve the equation:

$$x^2 + 6x + 25 = 0$$

→ You can use one of two methods for this

→ Either 'Completing the square' or the Quadratic formula

a = 1

b = 6

c = 25

<u>The Quadratic formula</u>

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sub in values

$$x = \frac{-6 \pm \sqrt{(6)^2 - (4 \times 1 \times 25)}}{2(1)}$$

Calculate the part under the root sign

$$x = \frac{-6 \pm \sqrt{-64}}{2}$$

Split it up

$$x = \frac{-6 \pm \sqrt{64}\sqrt{-1}}{2}$$

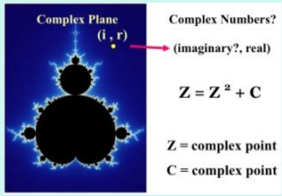
Simplify the roots

$$x = \frac{-6 \pm 8i}{2}$$

Divide all by 2

$$x = -3 \pm 4i$$

If the x^2 coefficient is greater than 1, or the x term is odd, the Quadratic formula will probably be the easiest method!



Complex Numbers

You can use both real and imaginary numbers to solve equations

Simplify each of the following, giving your answers in the form:

$$a + bi$$

where:

$$a \in R \text{ and } b \in R$$

This means a and b are real numbers

1) $(2 + 5i) + (7 + 3i)$

$= 9 + 8i$

Group terms together

2) $(2 - 5i) - (5 - 11i)$

$= 2 - 5i - 5 + 11i$

$= -3 + 6i$

'Multiply out' the bracket

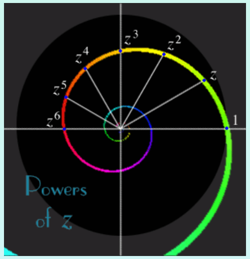
Group terms

3) $6(1 + 3i)$

$= 6 + 18i$

Multiply out the bracket

Complex Numbers



You can multiply complex numbers and simplify powers of I

Complex numbers can be multiplied using the same techniques as used in algebra.

You can also use the following rule to simplify powers of i:

$$i = \sqrt{-1}$$

$$i^2 = -1$$

<u>Multiply out the following bracket</u>

$$(2 + 3i)(4 + 5i)$$

Multiply put like you would algebraically (eg) grid method, FOIL, smiley face etc)

$$= 8 + 12i + 10i + 15i^2$$

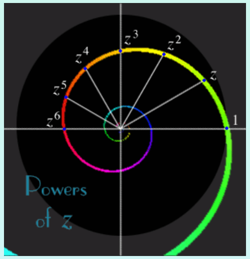
Group i terms, write i^2 as -1

$$= 8 + 22i + 15(-1)$$

Simplify

$$= -7 + 22i$$

Complex Numbers



You can multiply complex numbers and simplify powers of I

Complex numbers can be multiplied using the same techniques as used in algebra.

You can also use the following rule to simplify powers of i:

$$i = \sqrt{-1}$$

$$i^2 = -1$$

<u>Express the following in the form a + bi</u>

$$(7-4i)^2$$

Write as a double bracket

$$= (7-4i)(7-4i)$$

Multiply out

$$= 49 - 28i - 28i + 16i^2$$

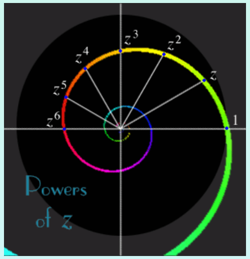
Group i terms, write i^2 as -1

$$= 49 - 56i + 16(-1)$$

Simplify

$$= 33 - 56i$$

Complex Numbers



You can multiply complex numbers and simplify powers of I

Complex numbers can be multiplied using the same techniques as used in algebra.

You can also use the following rule to simplify powers of i:

$$i = \sqrt{-1}$$

$$i^2 = -1$$

Simplify the following:

$(2-3i)(4-5i)(1+3i)$

$(2-3i)(4-5i)$

$= 8 - 12i - 10i + 15i^2$

$= 8 - 22i + 15(-1)$

$= -7 - 22i$

Start with the first 2 brackets

Multiply out

Group i terms, replace i^2 with -1

Simplify

Now multiply this by the 3rd bracket

$(-7-22i)(1+3i)$

$= -7 - 22i - 21i - 66i^2$

$= -7 - 43i - 66(-1)$

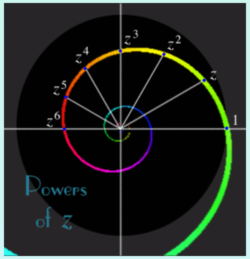
$= 59 - 43i$

Multiply out the brackets

Group i terms and replace i^2 with -1

Simplify

Complex Numbers



You can multiply complex numbers and simplify powers of I

Complex numbers can be multiplied using the same techniques as used in algebra.

You can also use the following rule to simplify powers of i:

$$i = \sqrt{-1}$$

$$i^2 = -1$$

<u>Simplify:</u>

1) i^3

Split up

$= i^2 \times i$

Replace i^2 with -1

$= -1 \times i$

Simplify

$= -i$

2) i^4

Split up

$= i^2 \times i^2$

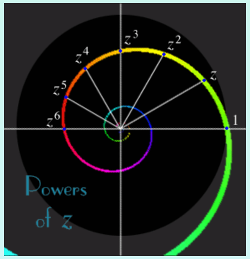
Replace the i^2 terms with -1

$= -1 \times -1$

Simplify

$= 1$

Complex Numbers



You can multiply complex numbers and simplify powers of I

Complex numbers can be multiplied using the same techniques as used in algebra.

You can also use the following rule to simplify powers of i:

$$i = \sqrt{-1}$$

$$i^2 = -1$$

Simplify:

3) $(2i)^5$

Write both as a power of 5

$= 2^5 \times i^5$

Split up the i terms

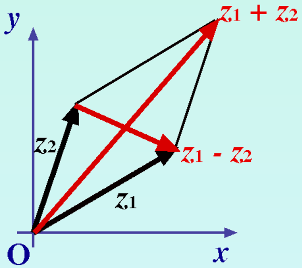
$= 2^5 \times i^2 \times i^2 \times i$

Work out 2^5 and replace the i^2 terms

$= 32 \times -1 \times -1 \times i$

Simplify

$= 32i$



Complex Numbers

You can find the complex conjugate of a complex number

You can write down the complex conjugate of a complex number, and it helps you divide one complex number by another

If a complex number is given by:
a + bi

Then the complex conjugate is:
a – bi

(You just reverse the sign of the imaginary part!)

Together, these are known as a **complex conjugate pair**

The complex conjugate of z is written as z*

Write down the complex conjugate of:

a) $2+3i$

$=2-3i$

Reverse the sign of the imaginary term

b) $5-2i$

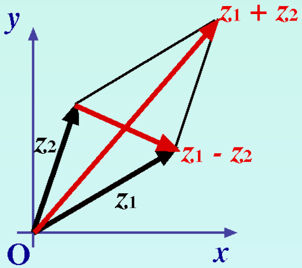
$=5+2i$

Reverse the sign of the imaginary term

c) $1-i\sqrt{5}$

$=1+i\sqrt{5}$

Reverse the sign of the imaginary term



Complex Numbers

You can find the complex conjugate of a complex number

Find z + z*, and zz*, given that:

z = 2 – 7i

→ z* = 2 + 7i

$z + z^*$

$= (2 - 7i) + (2 + 7i)$

$= 4$

Replace z and z*

Group terms

zz^*

$= (2 - 7i)(2 + 7i)$

$= 4 + 14i - 14i - 49i^2$

$= 4 - 49(-1)$

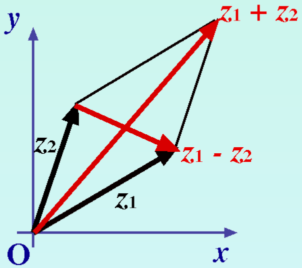
$= 53$

Replace z and z*

Multiply out

The i terms cancel out, replace i^2 with -1

Simplify



Complex Numbers

You can find the complex conjugate of a complex number

Find z + z*, and zz*, given that:

z = 2√2 + i√2

→ z* = 2√2 - i√2

$$z + z^*$$

$$= \left(2\sqrt{2} + i\sqrt{2}\right) + \left(2\sqrt{2} - i\sqrt{2}\right)$$

$$= 4\sqrt{2}$$

Replace z and z*

Group terms

$$zz^*$$

$$= \left(2\sqrt{2} + i\sqrt{2}\right)\left(2\sqrt{2} - i\sqrt{2}\right)$$

$$= 4\sqrt{4} + 2i\sqrt{4} - 2i\sqrt{4} - i^2\sqrt{4}$$

$$= 8 - (-1)(2)$$

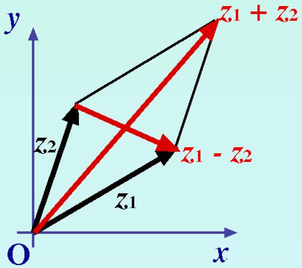
$$= 10$$

Replace z and z*

Multiply out

Some terms cancel out, replace i^2 with -1

Simplify



Complex Numbers

You can find the complex conjugate of a complex number

Simplify:

$(10+5i) \div (1+2i)$

With divisions you will need to write it as a fraction, then multiply both the numerator and denominator by the complex conjugate of the denominator

(This is effectively the same as rationalising when surds are involved!)

$$\frac{10+5i}{1+2i} \times \frac{1-2i}{1-2i}$$

Multiply by the complex conjugate of the denominator

$$= \frac{(10+5i)(1-2i)}{(1+2i)(1-2i)}$$

Expand both brackets

$$= \frac{10+5i-20i-10i^2}{1+2i-2i-4i^2}$$

Group i terms, replace the i^2 terms with -1 (use brackets to avoid mistakes)

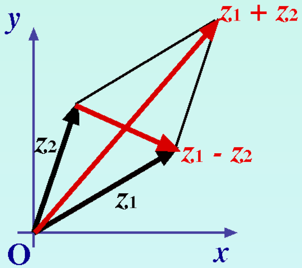
$$= \frac{10-15i-10(-1)}{1-4(-1)}$$

Simplify terms

$$= \frac{20-15i}{5}$$

Divide by 5

$$= 4-3i$$



Complex Numbers

You can find the complex conjugate of a complex number

Simplify:

$(5 + 4i) \div (2 - 3i)$

With divisions you will need to write it as a fraction, then multiply both the numerator and denominator by the complex conjugate of the denominator

(This is effectively the same as rationalising when surds are involved!)

$$\frac{5+4i}{2-3i} \times \frac{2+3i}{2+3i}$$

Multiply by the complex conjugate of the denominator

$$= \frac{(5+4i)(2+3i)}{(2-3i)(2+3i)}$$

Expand both brackets

$$= \frac{10+8i+15i+12i^2}{4+6i-6i-9i^2}$$

Group i terms, replace the i^2 terms with -1 (use brackets to avoid mistakes)

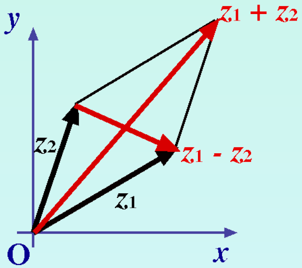
$$= \frac{10+23i+12(-1)}{4-9(-1)}$$

Simplify terms

$$= \frac{-2+23i}{13}$$

Split into two parts (this is useful for later topics!)

$$= -\frac{2}{13} + \frac{23}{13}i$$



Complex Numbers

You can find the complex conjugate of a complex number

If the roots a and b of a quadratic equation are complex, a and b will always be a complex conjugate pair

→ You can find what a quadratic equation was by using its roots

→ Let us start by considering a quadratic equation with real solutions…

$$x^2 + 7x + 10 = 0$$

Factorise

$$(x+5)(x+2) = 0$$

Solve

$$Roots\ are\ -5\ and\ -2$$

<u>Add the roots together</u>

$$(-5) + (-2)$$

$$= -7$$

Adding the roots gives the <u>negative</u> of the 'b' term

<u>Multiply the roots</u>

$$(-5) \times (-2)$$

$$= 10$$

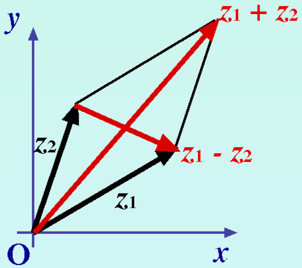
Multiplying the roots gives the 'c' term

This will work every time!

→ If you have the roots of a quadratic equation:

→ Add them and reverse the sign to find the 'b' term

→ Multiply them to find the 'c' term



Complex Numbers

You can find the complex conjugate of a complex number

If the roots a and b of a quadratic equation are complex, a and b will always be a complex conjugate pair

→ You can find what a quadratic equation was by using its roots

→ Let us start by considering a quadratic equation with real solutions…

$$x^2 + 2x - 24 = 0$$

Factorise

$$(x+6)(x-4) = 0$$

Solve

$$Roots\ are\ -6\ and\ 4$$

<u>Add the roots together</u>

$$(-6) + (4)$$

$$= -2$$

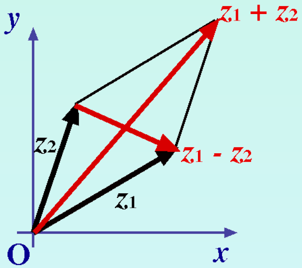
Adding the roots gives the <u>negative</u> of the 'b' term

<u>Multiply the roots</u>

$$(-6) \times (4)$$

$$= -24$$

Multiplying the roots gives the 'c' term



Complex Numbers

You can find the complex conjugate of a complex number

Find the quadratic equation that has roots 3 + 5i and 3 – 5i

<u>Add the roots together</u>

$(3+5i)+(3-5i)$

Group terms

$=6$

$So\ the\ 'b'term\ is\ -6$

<u>Multiply the roots</u>

$(3+5i)(3-5i)$

Multiply out brackets

$=9+15i-15i-25i^2$

Group the 'i' terms, replace i^2 with -1

$=9-25(-1)$

Simplify

$=34$

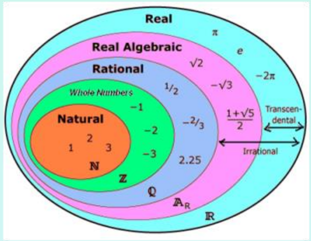
$So\ the\ 'c'term\ is\ 34$

Now you have the b and c coefficients, you can write the equation!

$The\ equation\ is\ therefore:$

$x^2-6x+34=0$

Complex Numbers



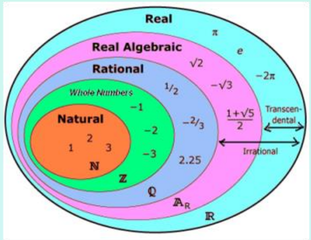
You can represent complex numbers on an Argand diagram

A grid where values for x and y can be plotted is known as a Cartesian set of axes (after Rene Descartes)

An Argand diagram is very similar, but the x-axis represents real numbers and the y-axis represents imaginary numbers.

Complex numbers can be plotted on an Argand diagram, by considering the real and imaginary parts as coordinates

Complex Numbers



You can represent complex numbers on an Argand diagram

Represent the following complex numbers on an Argand diagram:

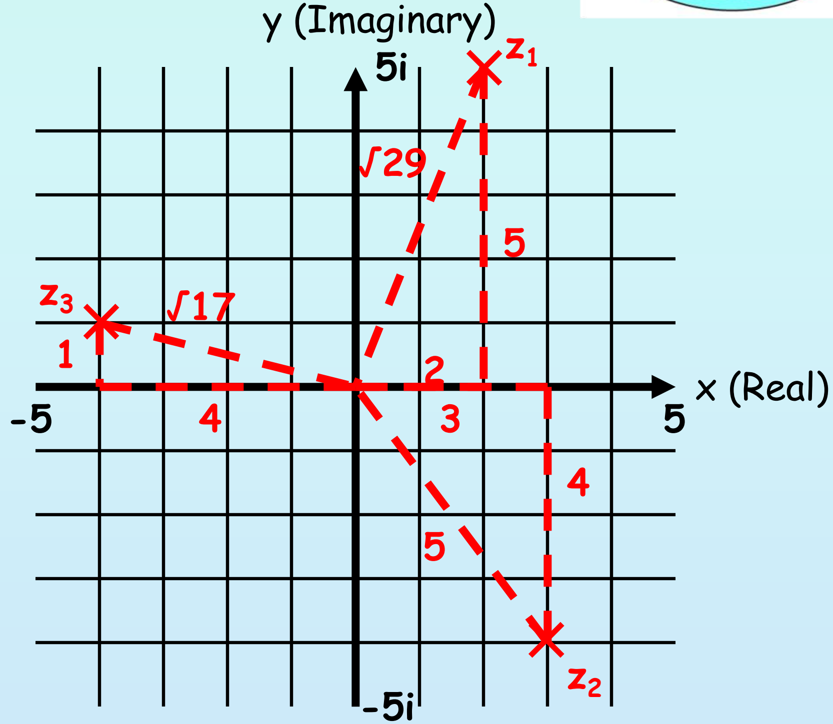
$$z_1 = 2 + 5i$$

$$z_2 = 3 - 4i$$

$$z_3 = -4 + i$$

Find the magnitude of $|OA|$, $|OB|$ and $|OC|$, where O is the origin of the Argand diagram, and A, B and C are z_1, z_2 and z_3 respectively

→ You can use Pythagoras' Theorem to find the magnitude of the distances



$$|OA| = \sqrt{2^2 + 5^2}$$

$$|OA| = \sqrt{29}$$

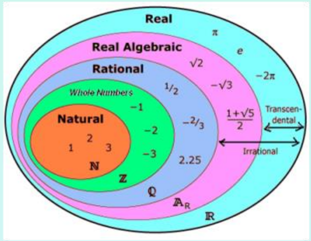
$$|OB| = \sqrt{3^2 + 4^2}$$

$$|OB| = 5$$

$$|OC| = \sqrt{4^2 + 1^2}$$

$$|OC| = \sqrt{17}$$

Complex Numbers



You can represent complex numbers on an Argand diagram

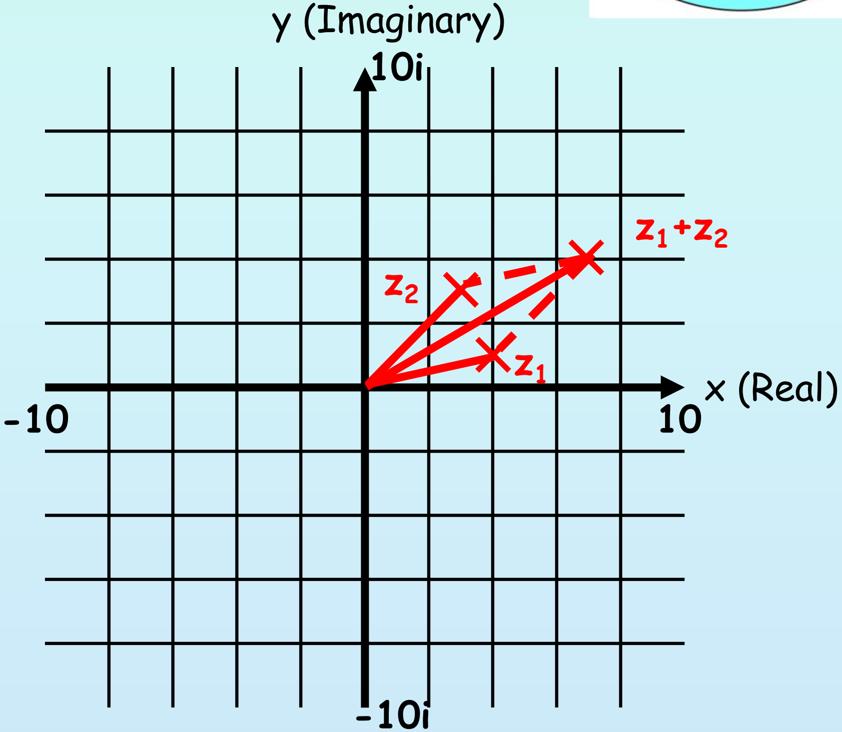
$z_1 = 4 + i$ $\qquad z_2 = 3 + 3i$

Show z_1, z_2 and $z_1 + z_2$ on an Argand diagram

$$z_1 + z_2$$

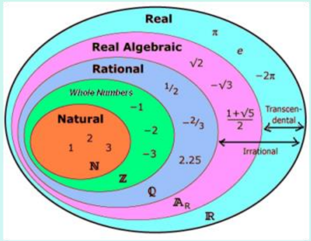
$$(4 + i) + (3 + 3i)$$

$$= 7 + 4i$$



Notice that vector $z_1 + z_2$ is effectively the diagonal of a parallelogram

Complex Numbers



You can represent complex numbers on an Argand diagram

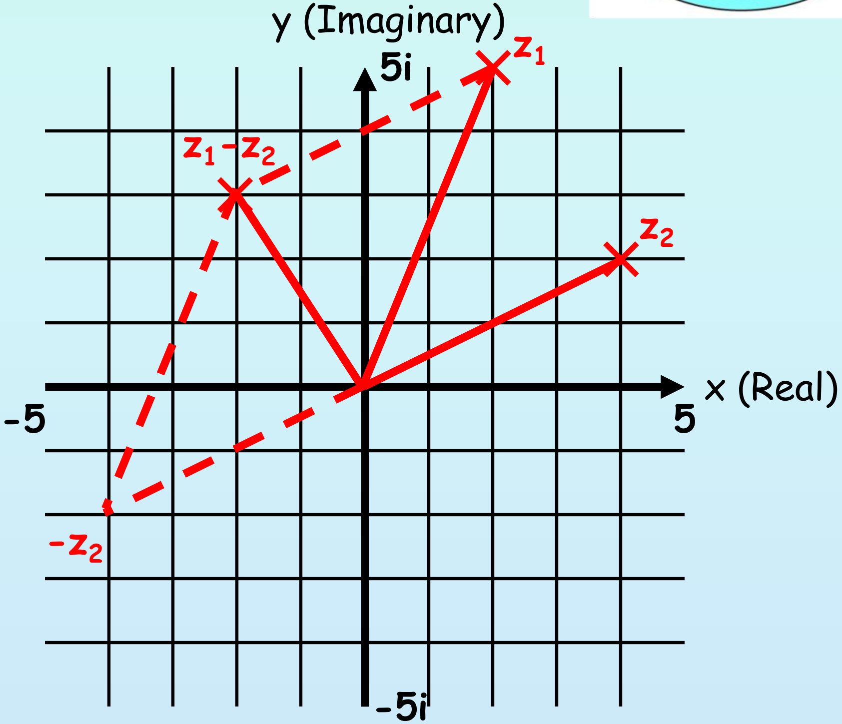
$z_1 = 2 + 5i \qquad z_2 = 4 + 2i$

Show z_1, z_2 and $z_1 - z_2$ on an Argand diagram

$$z_1 - z_2$$

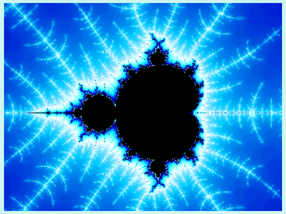
$$(2 + 5i) - (4 + 2i)$$

$$= -2 + 3i$$



Vector $z_1 - z_2$ is still the diagram of a parallelogram

→ One side is z_1 and the other side is $-z_2$ (shown on the diagram)



Complex Numbers

You can find the value of r, the modulus of a complex number z, and the value of θ, which is the argument of z

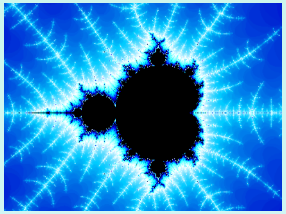
The modulus of a complex number is its magnitude – you have already seen how to calculate it by using Pythagoras' Theorem

→ It is usually represented by the letter r

The argument of a complex number is the angle it makes with the positive real axis

→ The argument is usually measured in radians

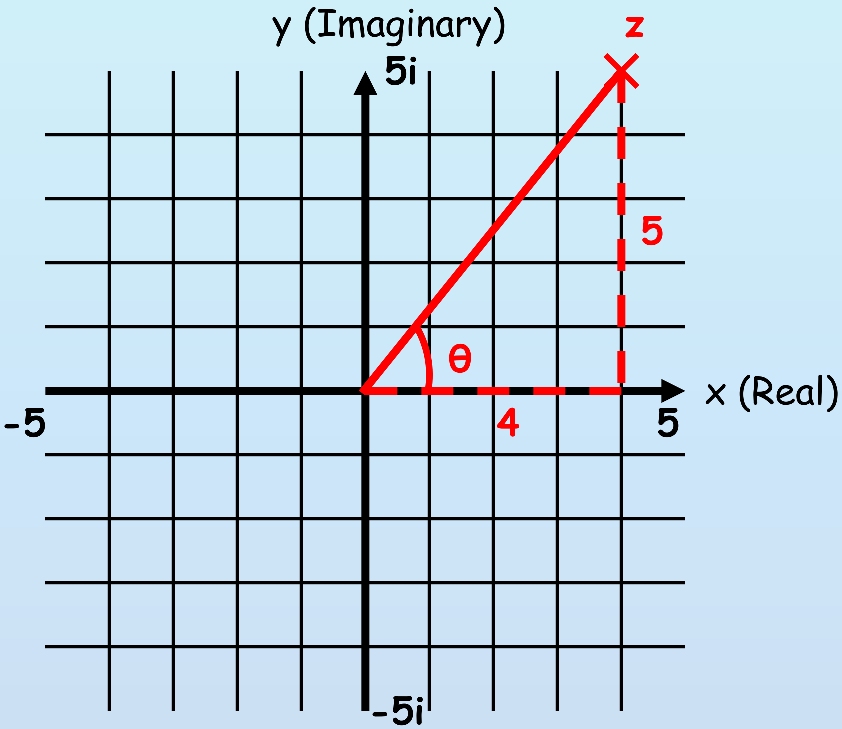
→ It will be negative if the complex number is plotted below the horizontal axis



Complex Numbers

You can find the value of r, the modulus of a complex number z, and the value of θ, which is the argument of z

Find, to two decimal places, the modulus and argument of z = 4 + 5i



Use Pythagoras' Theorem to find r

$$r = \sqrt{4^2 + 5^2}$$

Calculate

$$r = \sqrt{41}$$

Work out as a decimal (if needed)

$$r = 6.40\ (2dp)$$

Use Trigonometry to find arg z

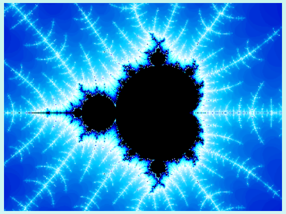
$$Tan\theta = \frac{O}{A}$$

Sub in values

$$Tan\theta = \frac{5}{4}$$

Calculate in radians

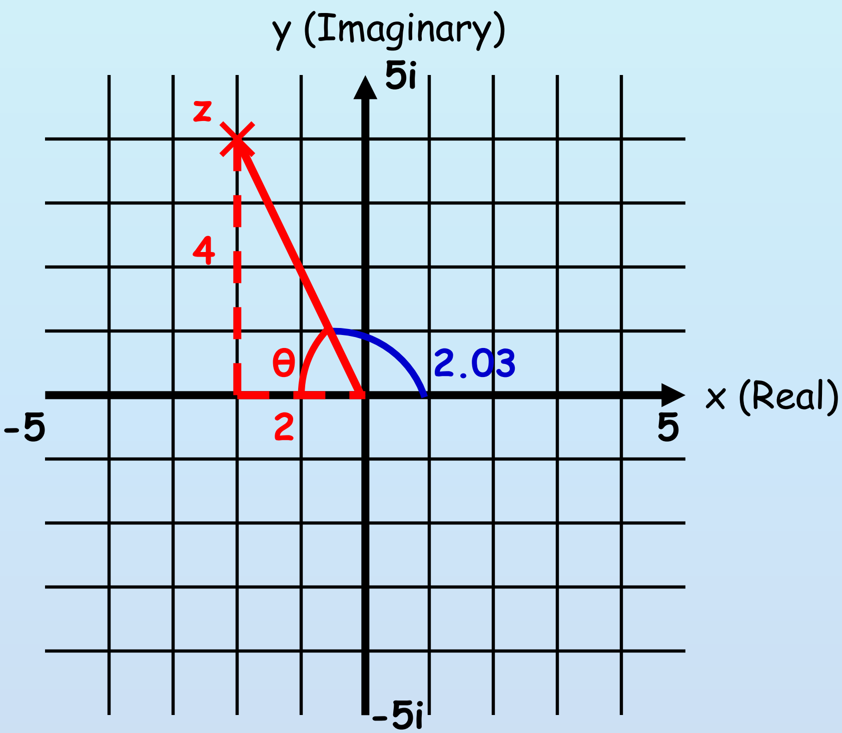
$$\theta = 0.90\ radians\ (2dp)$$



Complex Numbers

You can find the value of r, the modulus of a complex number z, and the value of θ, which is the argument of z

Find, to two decimal places, the modulus and argument of z = -2 + 4i



Use Pythagoras' Theorem to find r

$$r = \sqrt{2^2 + 4^2}$$

Calculate

$$r = \sqrt{20}$$

Work out as a decimal (if needed)

$$r = 4.47\ (2dp)$$

Use Trigonometry to find arg z

$$Tan\theta = \frac{O}{A}$$

Sub in values

$$Tan\theta = \frac{4}{2}$$

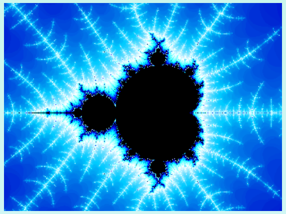
Calculate in radians

$$\theta = 1.11\ radians\ (2dp)$$

Subtract from π to find the required angle (remember π radians = 180°)

$$\pi - 1.11 = 2.03\ radians$$

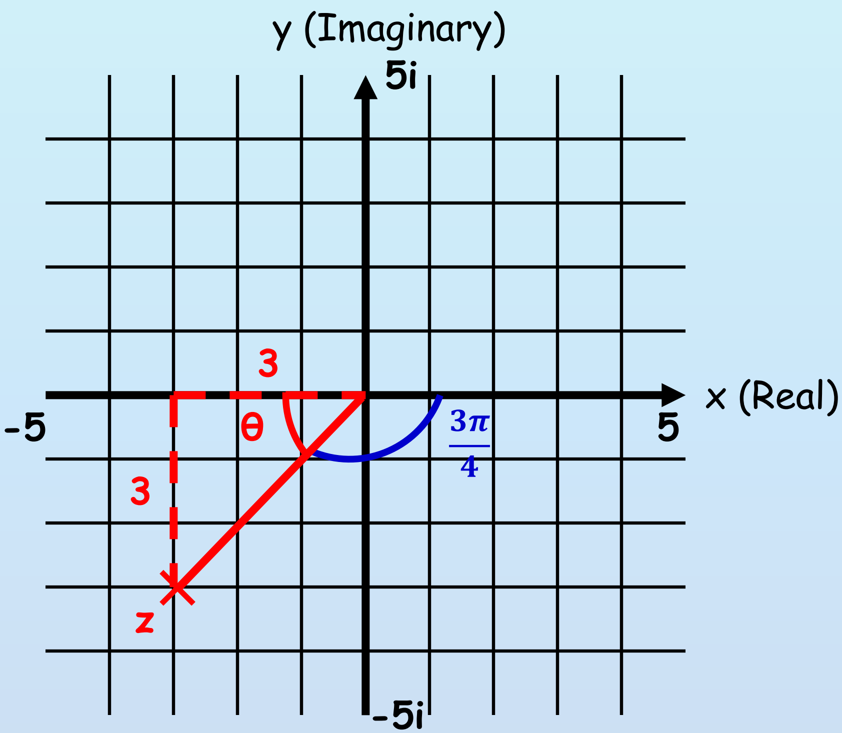
$$\arg z = 2.03$$



Complex Numbers

You can find the value of r, the modulus of a complex number z, and the value of θ, which is the argument of z

Find, to two decimal places, the modulus and argument of z = -3 - 3i



Use Pythagoras' Theorem to find r

$$r = \sqrt{3^2 + 3^2}$$

Calculate

$$r = \sqrt{18}$$

Work out as a decimal (if needed)

$$r = 4.24\ (2dp)$$

Use Trigonometry to find arg z

$$Tan\theta = \frac{O}{A}$$

Sub in values

$$Tan\theta = \frac{3}{3}$$

Calculate in radians

$$\theta = \frac{\pi}{4}\ radians\ (2dp)$$

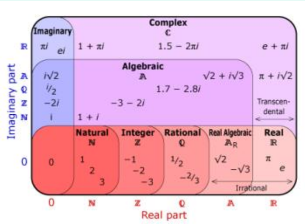
Subtract from π to find the required angle (remember π radians = 180°)

$$\pi - \frac{\pi}{4} = \frac{3\pi}{4} radians$$

As the angle is below the x-axis, its written as negative

$$\arg z = -\frac{3\pi}{4}$$

Complex Numbers



You can find the modulus-argument form of the complex number z

You have seen up to this point that a complex number z will usually be written in the form:

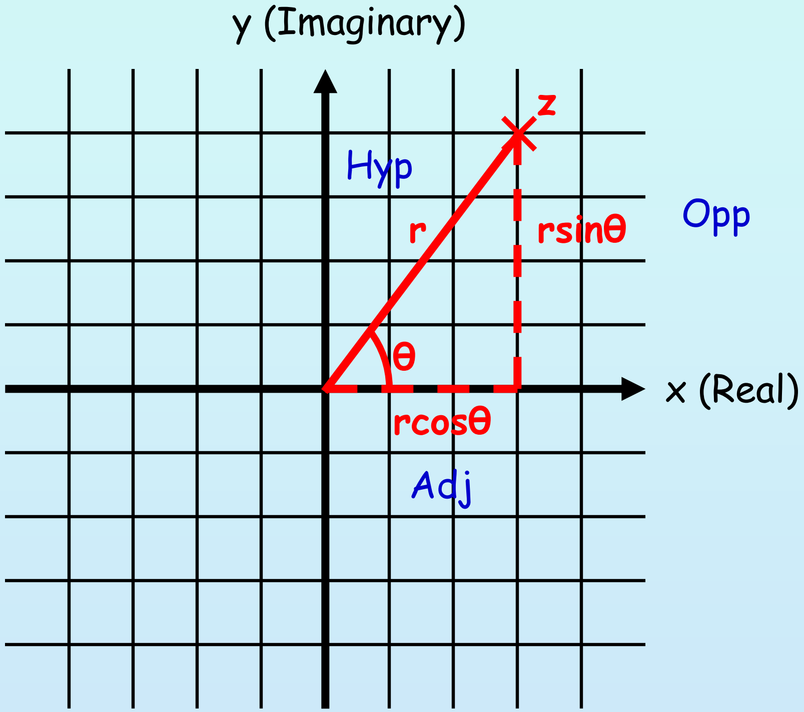
$$z = x + iy$$

The modulus-argument form is an alternative way of writing a complex number, and it includes the modulus of the number as well as its argument.

The modulus-argument form looks like this:

$$z = r(cos\theta + isin\theta)$$

r is the modulus of the number
θ is the argument of the number



By GCSE Trigonometry:

O
S H $\longrightarrow$ $Opp = Hyp \times Sin\theta \quad = rsin\theta$

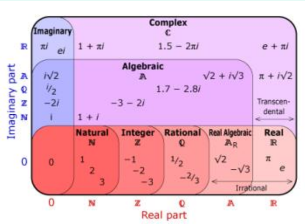
A
C H $\longrightarrow$ $Adj = Hyp \times Cos\theta \quad = rcos\theta$

$$z = rcos\theta + irsin\theta$$

Factorise

$$z = r(cos\theta + isin\theta)$$

$z = r(cos\theta + isin\theta)$



Complex Numbers

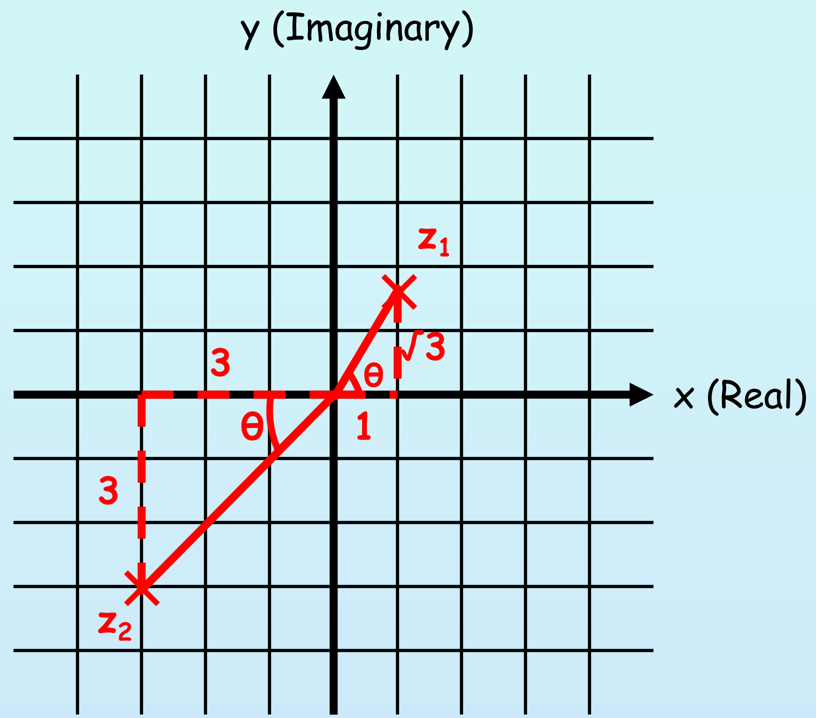
You can find the modulus-argument form of the complex number z

Express the numbers following numbers in the modulus argument form:

$$z_1 = 1 + i\sqrt{3}$$

$$z_2 = -3 - 3i$$

$$z_1 = 2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)$$



<u>Modulus for z_1</u>

$$\sqrt{1^2 + \sqrt{3}^2}$$

$$= 2$$

<u>Argument for z_1</u>

$$Tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

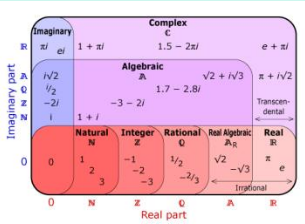
$$= \frac{\pi}{3}$$

$$z_1 = r(cos\theta + isin\theta)$$

$$z_1 = 2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)$$

$z = r(cos\theta + isin\theta)$

Complex Numbers



You can find the modulus-argument form of the complex number z

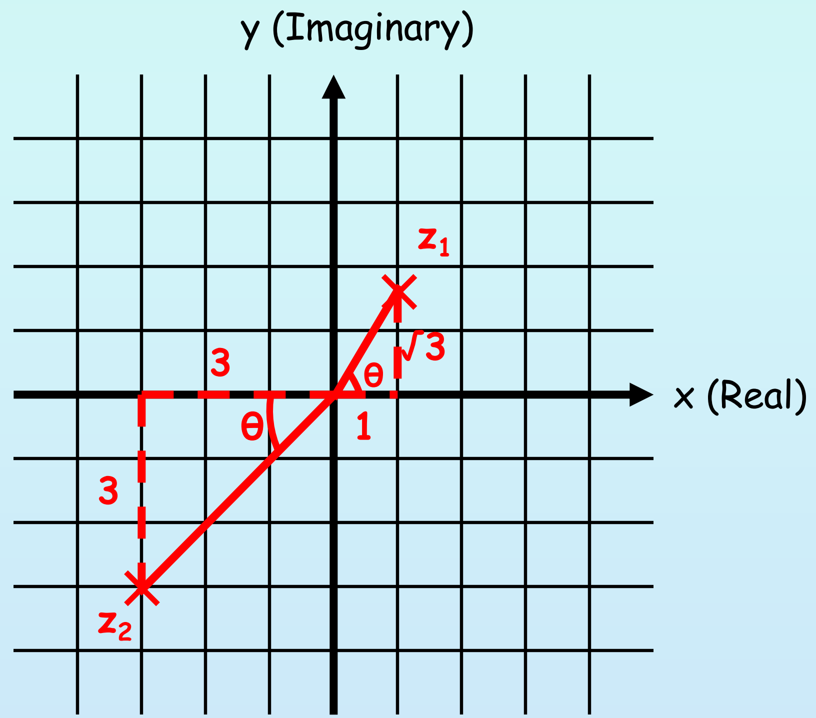
Express the numbers following numbers in the modulus argument form:

$$z_1 = 1 + i\sqrt{3}$$

$$z_2 = -3 - 3i$$

$$z_1 = 2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)$$

$$z_2 = 3\sqrt{2}\left(cos\left(-\frac{3\pi}{4}\right) + isin\left(-\frac{3\pi}{4}\right)\right)$$



Modulus for z_2

$$\sqrt{3^2 + 3^2}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

Argument for z_2

$$Tan^{-1}\left(\frac{3}{3}\right)$$

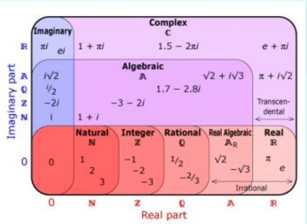
Remember the angle you actually want!

$$= \frac{\pi}{4} \longrightarrow = -\frac{3\pi}{4}$$

$$z_2 = r(cos\theta + isin\theta)$$

$$z_2 = 3\sqrt{2}\left(cos\left(-\frac{3\pi}{4}\right) + isin\left(-\frac{3\pi}{4}\right)\right)$$

$z = r(cos\theta + isin\theta)$



Complex Numbers

You can find the modulus-argument form of the complex number z

Express the numbers following numbers in the modulus argument form:

$$z_1 = 1 + i\sqrt{3}$$

$$z_2 = -3 - 3i$$

$$z_1 = 2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)$$

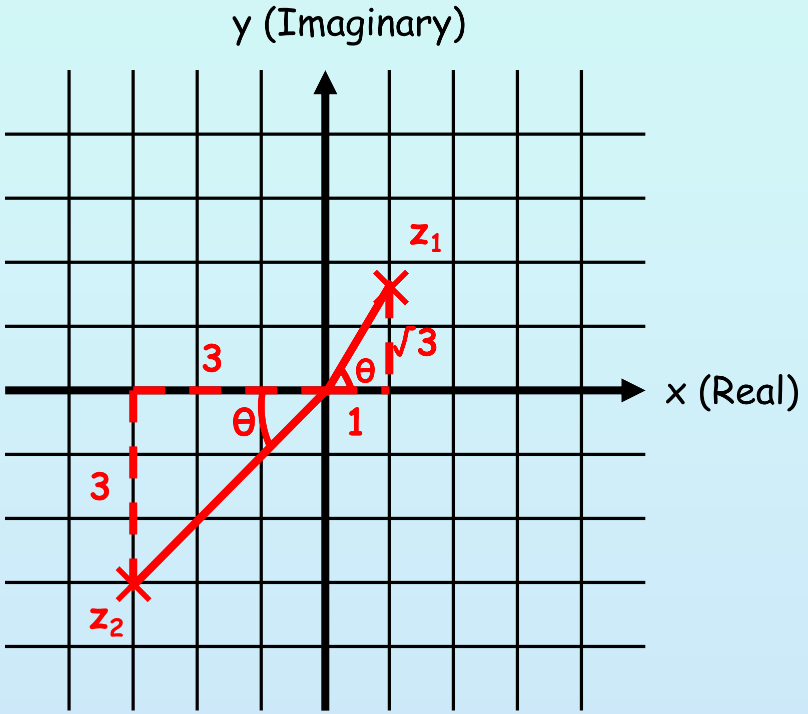
$$z_2 = 3\sqrt{2}\left(cos\left(-\frac{3\pi}{4}\right) + isin\left(-\frac{3\pi}{4}\right)\right)$$

Write down the value of $|z_1 z_2|$

$$|z_1 z_2| = |z_1||z_2|$$

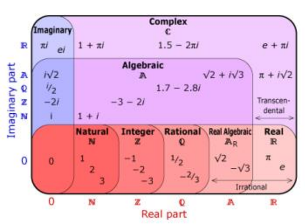
$$= 2 \times 3\sqrt{2}$$

$$= 6\sqrt{2}$$



$z = r(cos\theta + isin\theta)$

Complex Numbers



You can find the modulus-argument form of the complex number z

A complex number is represented in the modulus-argument form as:

$$z = 4\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)$$

Write the number in the form:

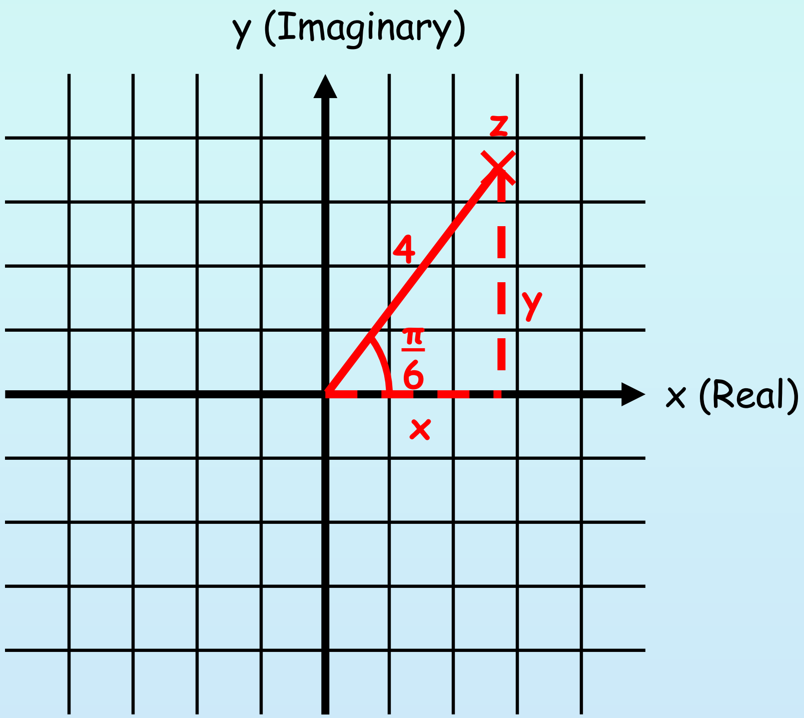
$$z = x + iy$$

Start by sketching the number on an Argand diagram

→ The modulus is 4

→ The angle is positive and less than $\pi/2$, so the point is somewhere in the top right section

→ Work out x and y using Trigonometry…



$x = rcos\theta$

$x = 4cos\frac{\pi}{6}$

$x = 2\sqrt{3}$

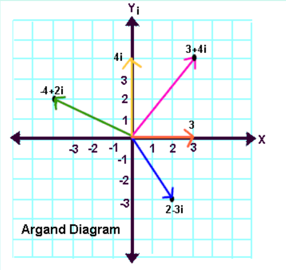
$y = rsin\theta$

$y = 4sin\frac{\pi}{6}$

$y = 2$

$So{:}\ \ z = 2\sqrt{3} + 2i$

Your sketch will help you decide whether answers are negative or positive!
It will also help you confirm what angle you should use…



Complex Numbers

You can solve problems involving complex numbers

Problems can be solved by equating the real and imaginary parts of a complex equation

→ This technique can also be used to square root a number

Given that:

$$3+5i=(a+ib)(1+i)$$

Find the real values of a and b

$$3+5i=(a+ib)(1+i)$$

Multiply out the bracket

$$3+5i=a+ai+bi+i^2b$$

Replace i²

$$3+5i=a+ai+bi+(-1)b$$

Remove the bracket

$$3+5i=a+ai+bi+-b$$

Move the real and imaginary terms together

$$3+5i=a-b+ai+bi$$

Factorise the imaginary terms

$$\boxed{3}+\boxed{5}i=\boxed{a-b}+i\boxed{(a+b)}$$

As the equations balance, the real and imaginary parts will be the same on each side

→ Compare them and form equations

1) $a-b=3$

2) $a+b=5$

Add the equations together

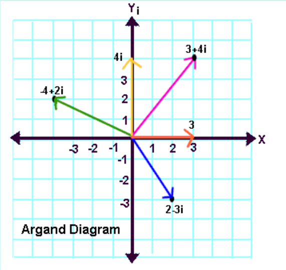
$$2a=8$$

Solve for a

$$a=4$$

Use a to find b

$$b=1$$



Complex Numbers

You can solve problems involving complex numbers

Problems can be solved by equating the real and imaginary parts of a complex equation

→ This technique can also be used to square root a number

Find the square roots of 3 + 4i

→ Let the square root of 3 + 4i be given by a + ib

$$\sqrt{3+4i} = a + ib$$

Square both sides

$$3 + 4i = (a+ib)^2$$

Write as a double bracket

$$3 + 4i = (a+ib)(a+ib)$$

Multiply out the bracket

$$3 + 4i = a^2 + abi + abi + i^2b^2$$

Move real terms and imaginary terms together

$$\boxed{3} + \boxed{4i} = \boxed{a^2 - b^2} + \boxed{2abi}$$

As the equations balance, the real and imaginary parts will be the same on each side

→ Compare them and form equations

1) $a^2 - b^2 = 3$

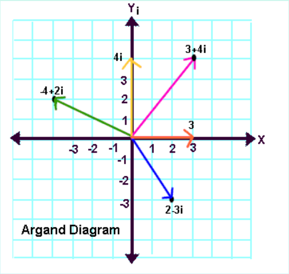
2) $2ab = 4$

Divide by 2

$$ab = 2$$

Divide by a

$$b = \frac{2}{a}$$



Complex Numbers

You can solve problems involving complex numbers

Problems can be solved by equating the real and imaginary parts of a complex equation

→ This technique can also be used to square root a number

Find the square roots of 3 + 4i

→ Let the square root of 3 + 4i be given by a + ib

$$a + ib$$

Use each pair of a and b to find the square roots

$$2 + i \qquad -2 - i$$

1) $a^2 - b^2 = 3$ 2) $\boxed{b = \frac{2}{a}}$

$$a^2 - \boxed{b^2} = 3$$

Replace b from the second equation

$$a^2 - \boxed{\left(\frac{2}{a}\right)^2} = 3$$

Square the bracket

$$a^2 - \frac{4}{a^2} = 3$$

Multiply each term by a^2

$$a^4 - 4 = 3a^2$$

Subtract $3a^2$

$$a^4 - 3a^2 - 4 = 0$$

You can factorise this like you would a quadratic

$$(a^2 - 4)(a^2 + 1) = 0$$

Each bracket can give solutions

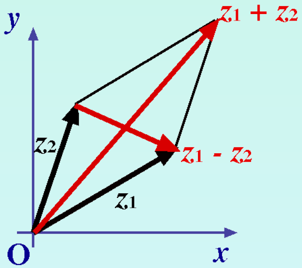
$$a^2 = 4 \; or \; \cancel{a^2 = -1}$$

But we want the real one so ignore $x^2 = -1$

$$a = 2 \; or \; -2$$

Use these to find their corresponding b values

$$b = 1 \; or \; -1$$



REMINDER FROM BEFORE

Complex Numbers

You can find the complex conjugate of a complex number

If the roots a and b of a quadratic equation are complex, a and b will always be a complex conjugate pair

→ You can find what a quadratic equation was by using its roots

→ Let us start by considering a quadratic equation with real solutions…

$x^2 + 7x + 10 = 0$

Factorise

$(x + 5)(x + 2) = 0$

Solve

$Roots\ are\ -5\ and\ -2$

Add the roots together

$(-5) + (-2)$

$= -7$

Adding the roots gives the negative of the 'b' term

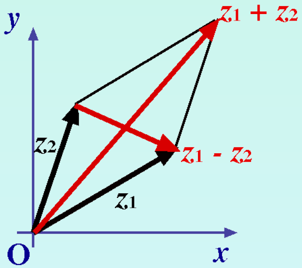
Multiply the roots

$(-5) \times (-2)$

$= 10$

Multiplying the roots gives the 'c' term

This will work every time!
→ If you have the roots of a quadratic equation:
→ Add them and reverse the sign to find the 'b' term
→ Multiply them to find the 'c' term



REMINDER FROM BEFORE

Complex Numbers

You can find the complex conjugate of a complex number

Find the quadratic equation that has roots 3 + 5i and 3 – 5i

Add the roots together

$(3+5i)+(3-5i)$

Group terms

$=6$

$So\ the\ 'b'term\ is\ -6$

Multiply the roots

$(3+5i)(3-5i)$

Multiply out brackets

$=9+15i-15i-25i^2$

Group the 'i' terms, replace i^2 with -1

$=9-25(-1)$

Simplify

$=34$

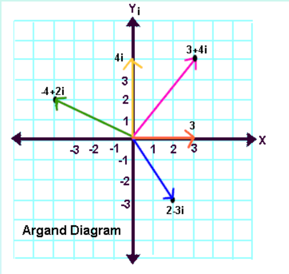
$So\ the\ 'c'term\ is\ 34$

Now you have the b and c coefficients, you can write the equation!

$The\ equation\ is\ therefore:$

$x^2-6x+34=0$

Complex Numbers



You can solve some types of polynomial equation with real coefficients

You have seen that if the roots of an equation are complex, they occur as a complex conjugate pair

If you know one complex root of a quadratic equation, you can find the whole equation itself

7 + 2i is one of the roots of a quadratic equation with real coefficients. Find the equation.

→ You can use a method from earlier in the chapter for this type of question

→ If 7 + 2i is one root, the other must be 7 – 2i

$7+2i \qquad 7-2i$

Add them together

$(7+2i)+(7-2i)$

$=14$

$So\ the\ 'b'term\ is\ -14$

Multiply them

$(7+2i)(7-2i)$

$=49+14i-14i-4i^2$

$=49-4(-1)$

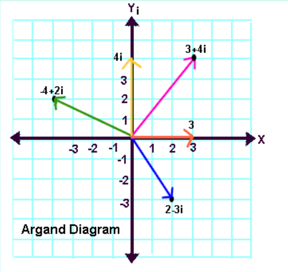
$=53$

$So\ the\ 'c'term\ is\ 53$

Now you know b and c you can write the equation

$$x^2-14x+53=0$$

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Show that x = 2 is a solution of the cubic equation:

$$x^3 - 6x^2 + 21x - 26 = 0$$

Hence, solve the equation completely.

$$x^3 - 6x^2 + 21x - 26 = 0$$

Sub in x = 2

$$(2)^3 - 6(2)^2 + 21(2) - 26 = 0$$

Calculate terms

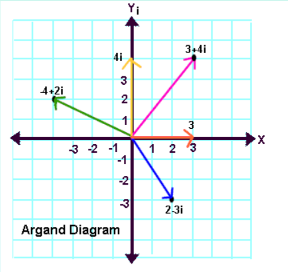
$$8 - 24 + 42 - 26 = 0$$

The equation balances

$$0 = 0$$

As subbing in a value of 2 makes the equation balance, x = 2 must be a solution

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Show that x = 2 is a solution of the cubic equation:

$$x^3 - 6x^2 + 21x - 26 = 0$$

Hence, solve the equation completely.

→ As x = 2 is a solution, the equation must have (x – 2) as a factor

→ Divide the expression by (x – 2) in order to help factorise it

Divide x^3 by x

Multiply the divisor by the answer and write it beneath

Subtract this from the original equation

Now divide $-4x^2$ by x

Multiply the divisor by this and continue these steps until you're finished!

$$x^2 - 4x + 13$$

$$x - 2 \,\big)\, x^3 - 6x^2 + 21x - 26$$

$$x^3 - 2x^2 \qquad -$$

$$-4x^2 + 21x - 26$$

$$-4x^2 + 8x \qquad -$$

$$13x - 26$$

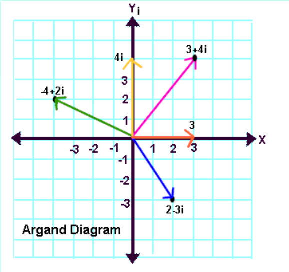
$$13x - 26 \qquad -$$

$$0$$

$$x^3 - 6x^2 + 21x - 26$$

$$= (x - 2)(x^2 - 4x + 13)$$

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Show that x = 2 is a solution of the cubic equation:

$$x^3 - 6x^2 + 21x - 26 = 0$$

Hence, solve the equation completely.

→ As x = 2 is a solution, the equation must have (x – 2) as a factor

→ Divide the expression by (x – 2) in order to help factorise it

$$x^3 - 6x^2 + 21x - 26 = 0$$

$$(x-2)(x^2 - 4x + 13) = 0$$

Either this bracket is 0

$$x - 2 = 0$$

$$x = 2$$

We already knew this solution!

Or this bracket is 0

$$x^2 - 4x + 13 = 0$$

Use completing the square

$$(x-2)^2 + 9 = 0$$

Subtract 9

$$(x-2)^2 = -9$$

Square root

$$x - 2 = \pm 3i$$

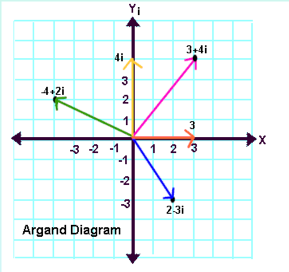
Add 2

$$x = 2 \pm 3i$$

The solutions of the equation $x^3 - 6x^2 + 21x - 26 = 0$ are:

$$x = 2 \quad x = 2 + 3i \quad and \quad x = 2 - 3i$$

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Given that -1 is a root of the equation:

$$x^3 - x^2 + 3x + k = 0$$

Find the other two roots of the equation.

→ If we substitute -1 in, the equation will balance…

$$x^3 - x^2 + 3x + 5 = 0$$

$$x^3 - x^2 + 3x + k = 0$$

Sub in x = -1

$$(-1)^3 - (-1)^2 + 3(-1) + k = 0$$

Calculate each part

$$-1 - 1 - 3 + k = 0$$

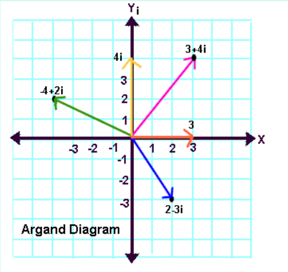
Rearrange to fin d k

$$k = 5$$

We now know the actual equation

$$x^3 - x^2 + 3x + 5 = 0$$

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Given that -1 is a root of the equation:

$$x^3 - x^2 + 3x + k = 0$$

Find the other two roots of the equation.

→ We can now solve the equation

$$x^3 - x^2 + 3x + 5 = 0$$

→ As -1 is a root, (x + 1) will be a factor of the equation

Divide x^3 by x

Multiply the divisor by the answer and write it beneath

Subtract this from the original equation

Now divide $-2x^2$ by x

Multiply the divisor by this and continue these steps until you're finished!

$$x^2 - 2x + 5$$

$$x + 1 \,\big)\, x^3 - x^2 + 3x + 5$$

$$x^3 + x^2 \quad -$$

$$-2x^2 + 3x + 5$$

$$-2x^2 - 2x \quad -$$

$$5x + 5$$

$$5x + 5 \quad -$$

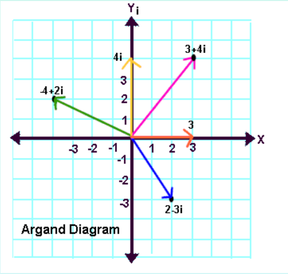
$$0$$

$$x^3 - x^2 + 3x + 5$$

$$= (x + 1)(x^2 - 2x + 5)$$

1H

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Given that -1 is a root of the equation:

$$x^3 - x^2 + 3x + k = 0$$

Find the other two roots of the equation.

→ We can now solve the equation

$$x^3 - x^2 + 3x + 5 = 0$$

→ As -1 is a root, (x + 1) will be a factor of the equation

$$(x+1)(x^2 - 2x + 5) = 0$$

$$x^3 - x^2 + 3x + 5 = 0$$

$$(x+1)(x^2 - 2x + 5) = 0$$

Either this bracket is 0

$$x + 1 = 0$$

$$x = -1$$

We already knew this solution!

Or this bracket is 0

$$x^2 - 2x + 5 = 0$$

Use completing the square

$$(x-1)^2 + 4 = 0$$

Subtract 4

$$(x-1)^2 = -4$$

Square root

$$x - 1 = \pm 2i$$

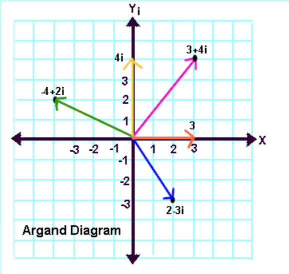
Add 1

$$x = 1 \pm 2i$$

The solutions of the equation $x^3 - x^2 + 3x + 5 = 0$ are:

$$x = -1 \quad x = 1 + 2i \quad and \quad x = 1 - 2i$$

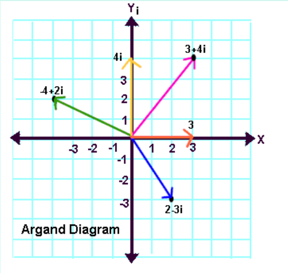
Complex Numbers



You can solve some types of polynomial equation with real coefficients

In a cubic equation, either:
→ All 3 solutions are real
→ One solution is real and the other 2 form a complex conjugate pair

Complex Numbers



You can solve some types of polynomial equation with real coefficients

You can also solve a quartic equation using this method

→ A quartic equation has an x power of 4, and will have a total of 4 roots

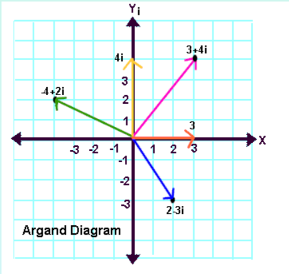
For a quartic equation, either:

→ All 4 roots are real

→ 2 roots are real and 2 are complex, forming a complex conjugate pair

→ All 4 roots are complex and form 2 complex conjugate pairs

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Given that 3 + i is a root of the quartic equation:

$$2x^4 - 3x^3 - 39x^2 + 120x - 50 = 0$$

Solve the equation completely.

As one root is 3 + i, we know that another root will be 3 – i

→ We can use these to find an expression which will factorise into the original equation

$$3 + i \qquad 3 - i$$

<u>Add them together</u>

$(3 + i) + (3 - i)$

$= 6$

$So\ the\ 'b'term\ is\ -6$

<u>Multiply them</u>

$(3 + i)(3 - i)$

$= 9 + 3i - 3i - i^2$

$= 9 - (-1)$

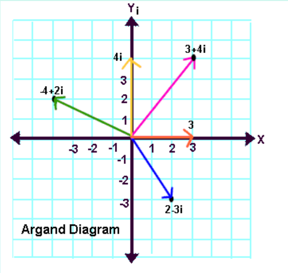
$= 10$

$So\ the\ 'c'term\ is\ 10$

Now you know b and c you can write an expression that will divide into the original equation

$$x^2 - 6x + 10$$

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Given that 3 + i is a root of the quartic equation:

$$2x^4 - 3x^3 - 39x^2 + 120x - 50 = 0$$

Solve the equation completely.

As one root is 3 + i, we know that another root will be 3 – i

→ We can use these to find an expression which will factorise into the original equation

$$x^2 - 6x + 10 \ is \ a \ factor$$

→ Divide the original equation by this!

$$2x^2 + 9x - 5$$

$$x^2 - 6x + 10 \,\big)\, 2x^4 - 3x^3 - 39x^2 + 120x - 50$$

$$2x^4 - 12x^3 + 20x^2$$

$$9x^3 - 59x^2 + 120x - 50$$

$$9x^3 - 54x^2 + 90x$$

$$-5x^2 + 30x - 50$$

$$-5x^2 + 30x - 50$$

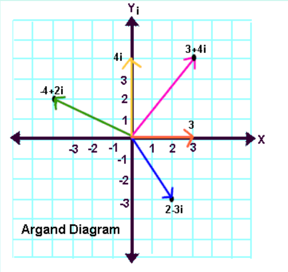
$$0$$

We have now factorised the original equation into 2 quadratics

$$2x^4 - 3x^3 - 39x^2 + 120x - 50$$

$$= (x^2 - 6x + 10)(2x^2 + 9x - 5)$$

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Given that 3 + i is a root of the quartic equation:

$$2x^4 - 3x^3 - 39x^2 + 120x - 50 = 0$$

Solve the equation completely.

As one root is 3 + i, we know that another root will be 3 – i

→ We can use these to find an expression which will factorise into the original equation

$$x^2 - 6x + 10 \ is \ a \ factor$$

→ Divide the original equation by this!

$$(x^2 - 6x + 10)(2x^2 + 9x - 5) = 0$$

We already have the solutions for this bracket!

We need to find the solutions for this one!

$$3 + i$$
$$3 - i$$

$$2x^2 + 9x - 5 = 0$$

Factorise

$$(2x + 1)(x - 5) = 0$$

$$x = -\frac{1}{2} \quad or \quad x = 5$$

$$2x^4 - 3x^3 - 39x^2 + 120x - 50 = 0$$

$$Solutions \ are: \quad x = 3 + i$$
$$x = 3 - i$$
$$x = -\frac{1}{2}$$
$$x = 5$$

All these will give the answer 0 when substituted in!

Summary

- You have been introduced to imaginary and complex numbers
- You have seen how these finally allow all quadratic equations to be solved
- You have learnt how to show complex numbers on an Argand diagram
- You have seen how to write the modulus-argument form of a complex number
- You have also seen how to solve cubic and quartic equations using complex numbers